Opisy przedmiotów do wyboru

wykłady monograficzne w języku angielskim

oferowane na stacjonarnych studiach II stopnia (magisterskich) dla 1 roku matematyki

semestr letni, rok akademicki 2017/2018

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1. Applications of the Theory of Functional Equations (wykład monograficzny w j. angielskim)

Specjalność Poziom 2 Status W
L. godz. tyg. 2 W + 2 K

Course outline:

- Applications in Geometry:
  1. Joint characterization of Euclidean, hyperbolic and elliptic geometries.
  2. Characterizations of the cross ratio.
  3. A description of certain subsemigroups of some Lie groups.
- Applications in Functional Analysis:
  1. Analytic form of linear-multiplicative functionals in the Banach algebra of integrable functions on the real line.
  2. A characterization of strictly convex spaces.
  3. Some new characterizations of inner product spaces.
  4. Birkhoff-James orthogonality.
  5. Addition theorems in Banach algebras; operator semigroups.

References

1. J. Aczel & J. Dhombres, *Functional equations in several variables*, Cambridge University Press, Cambridge, 1989.
2. J. Aczel & S. Gołąb, *Funktionalgleichungen der Theorie der Geometrischen Objekte*,PWN Warszawa, 1960.
3. J. Dhombres, *Some aspects of functional equations*, Chulalongkorn Univ., Bangkok, 1979.
4. D. Ilse, I. Lehman and W. Schulz, *Gruppoide und Funktionalgleichungen*, VEB Deutscher Verlag der Wissenschaften, Berlin, 1984.
5. M. Kuczma, *An introduction to the theory of functional equations and inequalities*, Polish Scientific Publishers & Silesian University, Warszawa-Kraków-Katowice, 1985.

Prowadzący: prof. dr hab. Roman Ger.

2. Borel Measures on Metric Spaces (wykład monograficzny w j. angielskim)

Specjalność Poziom 2 Status W
L. godz. tyg. 2 W + 2 K

Course outline:
Regularity of finite measures. Theorem of Ulam. Theorem of Riesz-Skorokhod. Riesz and Banach functionals. Fortet-Mourier norm. Weak convergence and theorem of Alexandrov. Theorem of Prokhorov. Convolution of measures. Christensen zero sets.

References

1. P. Billingsley, *Convergence of probability measures*, John Wiley & Sons 1999.
2. J.P.R. Christensen, *Topology and Borel structure*, North-Holland Mathematical Studies 10, North-Holland Publishing Company & American Elsevier Publishing Company 1974.
3. R.M. Dudley, *Real analysis and probability*, Cambridge studies in advanced mathematics 74, Cambridge University Press 2002.
4. I.I. Gikhman, A.V. Skorokhod, *The theory of stochastic processes. I*, Springer-Verlag 2004 [Russian original edition: Nauka, Moscow 1971].
5. A. Lasota, *Układy dynamiczne na miarach*, Wydawnictwo Uniwersytetu Śląskiego 2008.
6. M. Loeve, *Probability theory. I,* Graduate Texts in Mathematics 45, Springer-Verlag 1977.
7. St. Łojasiewicz, *Wstęp do teorii funkcji rzeczywistych*, Biblioteka Matematyczna 46, Państwowe Wydawnictwo Naukowe 1976. [English edition: An introduction to the theory of real functions, John Wiley & Sons 1988].
8. K.R. Parthasarathy, *Probability measures on metric spaces,* Academic Press 1967.

Prowadzący: prof. dr hab. Karol Baron.

3. **Fractal Geometry** (wykład monograficzny w j. angielskim)

Specjalność Poziom 2 Status W
L. godz. tyg. 2 W + 2 K

Course outline:

In geometry, one often investigates smooth objects like curves or surfaces. On the other hand, in the words of Benoit Mandelbrot:

> "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line."

A few decades ago, Mandelbrot coined the term fractal. At some point he mentioned:

> "Fractal Geometry is a new subject and each definition I try to give for it has turned out to be inappropriate."

Roughly speaking, fractal geometry studies geometric objects that are not smooth. An important concept is the one of dimension, being a measure for roughness. Here, dimensions do not need to be integers. There are many ways to generalize the concept of dimension to incorporate noninteger values. We are going to study a few of them: Hausdorff dimension, box-counting dimension, Minkowski dimension, and similarity dimension. During the course, we will learn about these dimensions, their properties and their relations. We will look at methods how to compute the dimensions and investigating properties of fractals and interactions between them.

References

1. The lecture will be mainly based on Kenneth Falconer's book "Fractal geometry. Mathematical foundations and applications" (there are by now three editions all by John Wiley & Sons, Ltd., Chichester published in 1990, 2003, and 2014, respectively; the part featuring in the lecture is mainly the same in all versions).

Prowadzący: dr Thomas Zürcher.

4. Lattices and Boolean Algebras (wykład monograficzny w j. angielskim)

Specjalność Poziom 2 Status W
L. godz. tyg. 2 W + 2 K

Course outline:

1. Boolean algebra as an algebraic structure, Boolean operations, connection with the theory of rings, homomorphism of a Boolean algebra (epimorphism, endomorphism) .
2. Order structures in Boolean algebras, lattice of regular open sets, ideals and filters.
3. Stone representation theorem, Boolean algebra as a field of sets, representation of a homomorphism as a continues functions.
4. Completion of a Boolean algebra.
5. Special classes of Boolean algebras (homogeneous Boolean algebras, rigid Boolean algebras, free Boolean algebras).

References

1. S. Koppelberg, Handbook of Boolean algebras, volume 1, North–Holland 1989,
2. R. Sikorski, Boolean Algebras, 2nd edn., Springer—Verlag 1964.

Prowadzący: Prof. dr hab. Aleksander Błaszczyk.

5. Selected Topics in Qualitative Theory of Differential Equations (wykład monograficzny w j. angielskim)

Specjalność Poziom 2 Status W
L. godz. tyg. 2 W + 2 K

Course outline:

This course serves as an introduction to the qualitative theory of ordinary differential equations. In particular, the following topics will be covered: notions of stability and instability, phase portraits of planar systems, Floquet theory of linear systems with periodic coefficients, conjugacies between linear systems with constant coefficients, hyperbolic critical points and topological conjugacies, Grobman-Hartman theorem, stable and unstable manifolds of a hyperbolic critical point, Hadamard-Perron theorem.

References

1. W.I. Arnold, *Równania różniczkowe zwyczajne*, PWN, Warszawa 1975.
2. L. Barreira, C. Valls, *Ordinary Differential Equations: Qualitative Theory*, American Mathematical Society, 2012.
3. C. Grant, *Theory of Ordinary Differential Equations*, CreateSpace Independent Publishing Platform, 2014.
4. J.K. Hale, *Ordinary Differential Equations*, Dover Publications, Mineola, 2009.
5. J.K. Hale, H. Koçak, *Dynamics and Bifurcations*, Springer-Verlag, New York, 1991.
6. J. Ombach, *Wykłady z równań różniczkowych wspomagane komputerowo - Maple*, Wydawnictwo Uniwersytetu Jagiellońskiego, Kraków, 1999.
7. A. Palczewski, *Równania różniczkowe zwyczajne*, Wydawnictwa Naukowo-Techniczne, Warszawa, 1999.
8. W. Walter, *Ordinary Differential Equations*, Springer-Verlag, New York, 1998.

Prowadzący: dr hab. Radosław Czaja.