STRONGLY CONVEX STOCHASTC PROCESSES

Dawid Kotrys

The definition of strongly convex stochastic processes is motivated by the definition of strongly convex functions, which was introduced by B. T. Polyak in 1966. Such functions play an important role in optimization theory and mathematical economics. In the last few years strongly convex functions have become once again the subject of intensive research. Many papers including new results were published. These works, as well as some earlier publications of K. Nikodem and A. Skowroński, were the main inspiration for my research.

The dissertation is divided into four chapters. The first one includes definitions, basic lemmas and theorems useful in the following parts of the dissertation. In the second chapter we present characterizations of convex and strongly convex stochastic processes by the use of the support functions, the first and the second derivative. We obtain also the counterparts of the classical Jensen inequality, the Hermite-Hadamard inequality and the Fejer inequality.

The third chapter is devoted to strongly Jensen-convex stochastic processes. We present there, in particular, some versions of the Jensen inequality, Kuhn type theorem, Bernstein-Doetsch theorem and Sierpiński theorem for strongly Jensen-convex stochastic processes.

The aim of the last chapter is to introduce the notion of strongly Wright-convex stochastic processes and to present some properties of them. In particular, we give there a characterization of strongly Wright-convex stochastic processes, which is a counterpart of the celebrated Ng's representation theorem of Wright-convex functions. We prove also a theorem on Jensen-convex stochastic processes majorized by Jensen--concave stochastic processes.