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SUMMARY OF THE PHD THESIS

ERGODIC PROPERTIES OF RANDOM DYNAMICAL SYSTEMS WITH JUMPS OCCURING WITH A STATE-DEPENDENT INTENSITY

In this PhD thesis, we analyze the asymptotics of the Markov operator, acting on Borel measures of a Polish space, determining the evolution of a distributions of time-homogeneous Markov chain, which describes the states immediately following the jumps of a certain piecewise-deterministic Markov process. Between any two consecutive jumps, this process evolves deterministically according to one of the semiflows, randomly selected among a finite number of available ones at the moment of jump. The jumps occur in a Poisson-like fashion with state-depentent rate, and each of them is attained by a continuous transformation of the pre-jump state, randomly drawn (with a place-dependent probability) from an arbitrarily given family of all possible ones.

Processes of this type (studied e.g. in [1–3, 7, 8]) are mainly used in biological models associated with gene expression (see [2,13]). As mentioned above, unlike the papers [1–3,7,8], the intensity of jumps in the model under consideration depends on current state of the system. The results contained in the above-mentioned papers include the case wherein the intervals between jumps have the same exponential distribution with a common constant parameter.

Our goal is to present two different methods which lead to establishing ergodicity of the Markov operator determining the transition law of our model in the Fortet–Mourier metric. Based on the concepts of non-expansiveness and semiconcentration, which play a key role, among others, in some papers of T. Szarek [15, 16], we prove that the operator under consideration is asymptotically stable. The part of the work devoted to this issue is an adaptation of the results obtained within the article [11]. In the second part of the thesis, which draws heavily on the article [4], we show (imposing a little bit stronger assumptions on the set of semiflows), that the convergence of the distributions of the chain to the unique invariant measure occurs at a geometric rate. We refer to this property by saying that the chain is exponential ergodic in the Forter–Mourier distance. For this purpose, we will use the coupling techniques, initiated by M. Hairer in [5] (and also developed in [9]). Appealing to A. Shirikyan's law of large numbers [14], we prove the strong law of large numbers for for our model.

The last chapter illustrates the application of the result obtained in Section 2.2 in the analysis of the exponential ergodicity of the Markov operator associated with a version of the Poisson driven stochastic differential equation, which generalizes the models examined, among others, by A. Lasota and J. Traple in [12], K. Horbacz in [6] and J. Kazak in [10].

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